Use of Radar Precipitation Measurements for Flash Flood Forecasting*

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Further development of mathematical models describing flash flood formation imposes higher requirements on the initial information needed for these models. For example, to use two-dimensional flash-flood formation models in practice one must know how to assign the fields of various hydrometeorological elements, and, primarily, how to construct detailed maps of spatial precipitation distribution over short time intervals. As a rule, the existing precipitation (pluviographic) network does not permit one to construct such maps with sufficient resolution in space and time. This possibility is potentially available in radar precipitation observations, which yield simultaneous data for a large area (20,000-30,000 km² for precipitation). The main difficulty is in interpreting the radar data on a "real time scale", i.e., directly at the instant of precipitation observation.

Many studies are devoted to the interpretation of radar information and there are fairly reliable methods for determining the amount of precipitation, based on various modifications of the radar storm-detection equation [6]. However, there are only a few foreign papers [9] on the problem involved in the use of this information in flash-flood computations (forecasts).

Therefore we will investigate only those problems that are associated with the use of already analyzed precipitation information in runoff computations from models allowing for the uneven spatial distribution of precipitation. These include the problem of computing precipitation in a given regular system of points, analysis of the effect of averaging the radar-measured amount of precipitation over space and time, and comparison of computations from radar and pluviograph data.

Another feature of radar measurements of the amount of precipitation must also be taken into account. Determination of radar reflectivity [6] and its conversion to precipitation for a large number of points simultaneously over short time intervals are very laborious. Therefore, the network of MRL-1 and MRL-2 radar sets yields qualitative information in the operating mode only on precipitation averaged strongly over the area, which is unsuitable for quantitative flash-flood forecasting. Attempts have been made in recent years to automate radar precipitation measurement and quan-

titative computation. For example, an automated system [2, 3] for obtaining the amount of precipitation over short time intervals (about 15 min) for 10-km² in a radius of 100 km has been developed at the Central Forecasting Institute on the basis of a MRL-2 radar set. A shortcoming of this system as regards operational flash-flood forecasting is that the processes of obtaining the information and its subsequent analysis are separated. The initial information on radar reflectivity is recorded on magnetic tape for a sufficiently long time interval (virtually over the entire rainfall), after which the tape is removed and inserted into a computer, where it is processed further and where precipitation at the nodes of the squares is computed. Consequently, to develop an operational radar system one must exclude the intermediate information carrier (magnetic tape) and apply the radar signals directly to the computer.

Such a modification should not lead to an essential change in the system for obtaining data on the amount of precipitation and in their accuracy. Therefore all our computations were performed from precipitation data obtained with the unmodified measuring system and made available to us by the staff of the Radar Laboratory of the Central Forecasting Institute. For comparative computations we also used hydrometeorological observations in the experimental drainage basin of the Medvenka River (the basin area upstream of the confluence of the Zakza River is 21.5 km²).

We used radar measurements of five rainfalls in the summer of 1974. Data from 15 radar stations (3.33 km apart) in the Medvenka basin area were used in the computations. Radar-measured precipitation was compared with pluviograph measurements at the Podmoskovnaya meteorological station, located near the outlet gaging station of the Medvenka basin (Fig. 1). Radar station 10 in Fig. 1 is closest to this meteorological station. However, considering the relatively low accuracy of coordinate control of the radar set and meteorological station, we compared the data also with precipitation averaged over four meteorological stations closest to the radar stations (in this case points 10, 11, 13, 14). Table 1 gives data on precipitation obtained by radar (in two variants) and pluviograph (Podmoskovnaya meteorological station).

These data show very good agreement between radar and pluviograph measurements, except at 8-9h on July 27, when total precipitation measured by radar over one hour exceeds the corresponding pluviograph value by a factor of 10. The relative errors of total precipitation during this rainfall are slightly smaller than the errors in the hourly values and do not exceed 15-20% in this case. The difference in the amount of precipitation averaged over four radar stations and the

Table 1

Comparison of Precipitation Measured by Radar and Pluviograph
in the Area of the Podmoskovnaya Meteorological Station in 1974,
mm

| Date	Time	Precipitation according to radar		Precipitation according to pluviograph
		for points (10, 11, 13, 14)	for point 10	
24/V	15—16h	2,72	2,77	2,3
	16—17h	2,36	2,40	2,4
	17—18h	2,38	2,47	2,1
	18—18h 40m	1,30	1,26	1,6
	15—18h 40m	8,76	8,90	8,4
27/VII	8—9h	2,56	2,76	0,3
	9—10h	1,39	1,42	0,8
	10—11h	8,05	8,71	8,5
	11—11h 35m	2,49	2,24	2,5
	8—11h 35m	14,49	15,13	12,1
	13h 20m—14h	1,89	1,83	2,3
	14—15h	1,00	0,97	0,9
	15—15h 45m	0,48	0,44	0
	13h 20m — 15h 45m	3,37	3,24	3,2
28/VII	18h 20m —20h 30m	0,39	0,30	0,2
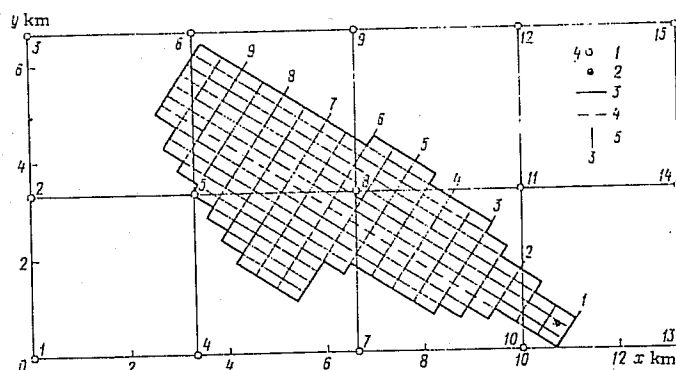
Fig. 1. Drainage basin of the Medvenka River upstream of
the mouth of the Zakza River with difference grid and near-
est radar stations (1).

2) Podmoskovnaya station; 3) boundary of basin; 4) straighten-
ed channel line; 5) boundary of particular areas.

Computation of the Amount of Precipitation at the Nodes of the Difference Grid

When runoff is computed from two-dimensional
models, the drainage basin is divided into many
rectangles by a system of lines parallel to the axes
of the coordinates. At the points of intersection of
these lines (nodes of the difference grid) one must
assign the initial information, particularly on pre-
cipitation intensity. The distance between nodes
depends on the topographic characteristics of the
drainage basin and runoff formation conditions, but
in most cases it does not exceed a few kilometers.
In the general case, the radar stations at which pre-
cipitation data are assigned may not coincide with
the nodes of the difference grid.

We will direct the axis of the Cartesian coordinates
parallel to the corresponding axes used in the pro-
cessing of radar data. We will plot the boundary of the
Medvenka River basin on the radar map. We will con-
struct a difference grid by dividing the basin by a sys-
tem of orthogonal straight lines (one of the system of
lines is parallel to the straightened channel line) into
rectangles with sides 500 × 250 m (Fig. 1). The idea is
to obtain from the radar points in the basin and its
vicinity the amount of precipitation at the nodes of the
grid, i.e., interpolate precipitation from one system
of regular points to another.

Methods of interpolation allowing for the spatial
structure of given fields are physically most justified.
In our case we will investigate precipitation fields
over very short time intervals (about 5-10 min). The
spatial structure of such fields has hardly been studied

and it can be assumed that because of the great variability of the characteristics of these fields in time and space, it will be impossible to obtain stable precipitation distributions over short time intervals. Therefore we will investigate two methods of interpolation that do not allow for the spatial structure of the precipitation field.

The first method is based on the characteristics of averaging over the area for radar data processing. Since the radar signals are averaged within a 3.3 × 3.3 km square upon transition from polar to Cartesian coordinates, we will assume that the amount of precipitation is constant for all points of the difference grid lying in this square and equal to the amount of precipitation measured by radar and referred to the center of the square. Consequently, in this case the precipitation data for the basin will be mosaical.

In the second approach we used linear interpolation. The amount of precipitation at the nodes of the difference grid was determined from four radar stations located at the angles of the square in which the given node was located. Using Fig. 1, we can write the following working relation for determining the amount of precipitation at a nodal point with the coordinates (x, y):

$$P(x, y) = P_v(1-\alpha)(1-\beta) + P_{v+1}(1-\alpha)\beta$$
$$+ P_{v+l}\alpha\beta + P_{v+l+1}\alpha(1-\beta),$$
$$\alpha = x/DR - \text{ent}\{x/DR\},$$
$$\beta = y/DR - \text{ent}\{y/DR\},$$
$$v = \text{ent}\{x/DR\}\,l + \text{ent}\{y/DR\} + 1,$$
(1)

where l is the number of lines parallel to the X-axis on which the radar measurements are assigned (in our case, $l = 3$); DR is the length of the side of a square, equal to 3.33 km; and P_v, P_{v+1}, P_{v+l}, P_{v+l+1} is the amount of precipitation measured by radar and referred to the points v, $v+1$, $v+l$, and $v+l+1$, respectively.

Comparison of the two interpolation methods showed that the amounts of precipitation, averaged for the entire basin, are very similar in the two cases. The average deviation does not exceed 1-2%. The point values of precipitation differ much more. For example, their standard deviation from long-period values at the nodes of the difference grid, computed by the two methods, amounts to 0.2 mm/hr (10-15%). The second interpolation method is preferable. Furthermore, numerical experiments with introduction of random errors (the method for assigning the errors will be described below) showed that the second method is less sensitive to the errors of measurement of the amount of precipitation. This is probably associated with some smoothing of errors when four points are used simultaneously.

Computation of Infiltration and of the Runoff Hydrograph

At each point of the difference grid we computed the rate of infiltration and of the water yield, as well as soil moisture distribution with depth from the following equations:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial\theta}{\partial z} - K\right),$$
(2)

$$J = -\left(D\frac{\partial\theta}{\partial z} - K\right)_{z=0},$$
(3)

$$q = P - J,$$
(4)

where θ is soil moisture by volume; D is the diffusion coefficient; K is hydraulic conductivity; J, q, and P are, respectively, the rates of infiltration and water yield and precipitation intensity; t is time; and z is depth.

The diffusion coefficient is associated with hydraulic conductivity by the relation

$$D = K\frac{\partial\psi}{\partial\theta},$$
(5)

where ψ is hydrostatic pressure, expressed in cm of water column.

In the general case, the hydrophysical soil characteristics (K, ψ, D) are functions of the coordinates and soil moisture. Considering the relative homogeneity of the soils of the Medvenka basin, we will assume that the hydrophysical characteristics depend only on soil moisture and will approximate these relations by the exponents:

$$K = K_m e^{n(\theta-\theta_m)},$$
(6)

$$\psi = -\psi_0 e^{-a\theta},$$
(7)

$$D = D_m e^{(n-a)(\theta-\theta_m)},$$
(8)

where θ_m is saturation moisture; K_m is maximum hydraulic conductivity for $\theta = \theta_m$; ψ_0 is maximum hydrostatic pressure for $\theta \to 0$; $D_m = K_m\psi_0 a e$ is the maximum diffusion coefficient for $\theta = \theta_m$; and n and a are parameters.

We determined the initial values of the parameters K_m, ψ_0, n, and a from the results of Sudnitsyn's experiments [7] for the Sod-Podzolic soils of the Rybinsk forestry. Their values proved to be: $K_m = 0.000,01$ cm/sec, $\psi_0 = 9,8 \cdot 10^6$ cm of water column; a = 28, and n = 55. Computations with these parameters showed that the computed runoff volumes exceed the actual volumes considerably. Therefore we refined the parameters K_m and ψ_0 and subsequently set $K_m = 0.000,05$ cm/sec and $\psi_0 = 9,8 \cdot 10^5$ cm of water column.

We computed the runoff hydrograph at the outlet gaging station from a simplified scheme, based on particular travel-time curves. The drainage basin was divided into nine particular areas by lines perpendicular to the river channel and spaced 1 km apart. For each area we determined the average depth of the water yield

$$\bar{q}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} q_j \Delta t,$$
(9)

where N_i is the number of grid notes in the i-th area and Δt is the step of integration with respect to time.

Then to compute the runoff hydrograph with allowance for (9) one can write the following relation:

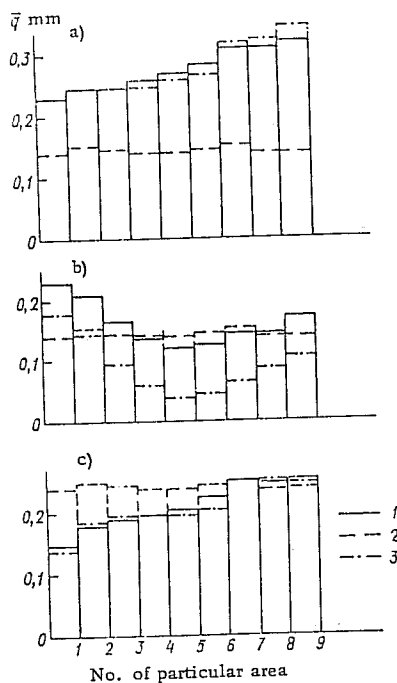$$Q(t) = \frac{1}{F}\sum_{i=1}^{9} F_i \int_0^t \bar{q}_i(\tau) P_i(t-\tau)\,d\tau,$$
(10)

Fig. 2. Variations in water yield for particular areas with precipitation averaged over 10-min intervals:

1) according to radar data (second interpolation method); 2) according to pluviograph data; 3) according to radar data (first interpolation method) Medvenka River, May 2, 1974; a) t = 18 hr; b) t = 17 hr; c) t = 16 hr.

where F is the basin area; F_i is the area of the i-th particular basin; and $P_i(t)$ is the travel-time curve for the i-th area. The travel-time curves were approximated by a gamma distribution, in which case the dimensional parameter was taken to be constant for all curves and the other parameter was increased by unity for each successive curve, starting from the mouth.

This way of allowing for irregularity in the computation of the runoff hydrograph is somewhat schematic, but when there is a large number of particular areas it makes it possible to estimate the effect of the irregularity of the water yield over the basin in the first approximation. It is difficult to compute the hydrograph from two-dimensional hydrodynamic models in greater detail because of the large volume of computations in numerical experiments.

Equations (2)-(4) were solved numerically using the explicit difference scheme, which is central in depth and directed forward in time. For any r-th point (r = 1, 2, ..., 210) one can write the following difference equations:

$$\frac{Q_j^{i+1} - Q_j^i}{\Delta t} = \frac{1}{\Delta z}\left\{ \widetilde{D}_{j,\,j+1}^i \frac{Q_{j+1}^i - Q_j^i}{\Delta z}\right.$$

$$\left. - \widetilde{D}_{j-1,\,j}^i \frac{Q_j^i - Q_{j-1}^i}{\Delta z} - \widetilde{K}_{j,\,j+1}^i + \widetilde{K}_{j-1,\,j}^i \right\}, \qquad (11)$$

$$J_r^{i+1} = - \widetilde{D}_{1,0}^{i+1} \frac{\theta_1^{i+1} - \theta_0^{i+1}}{\Delta z} + \widetilde{K}_{1,0}^{i+1}, \qquad (12)$$

$$q_r^{i+1} = P_r^{i+1} - J_r^{i+1}, \qquad (13)$$

where Δz is the step of integration with respect to depth; i and j are the indexes of the points of the grid along the t and z axes, respectively; and $\widetilde{D}_{j,\,j+1}^i$, and $\widetilde{K}_{j,\,j+1}^i$ are the values of the diffusion coefficient and of hydraulic conductivity, averaged for the points j, j + 1 at the instant of time i.

For the difference equations (11) and (12) we assigned the following initial and boundary conditions:

$$\text{for } i=0 \quad \theta_j^0 = \theta_0(j,\,r),$$

$$\text{for } j=0 \quad \theta_0^i = \theta_1^i - \Delta z \frac{\widetilde{K}_{0,1}^i - P_r^i}{\widetilde{D}_{0,1}^i}, \quad \text{if} \quad h_r^i = 0,$$

$$\theta_0^i = \theta_m, \qquad \text{if} \quad h_r^i \neq 0,$$

$$\text{for } j=L \quad \theta_L^i = \theta_0(L,\,r),$$

where h_r^i is the layer of water at the soil surface; L is the lower boundary of the soil volume singled out; and $\theta_0(j,\,r)$ is the initial soil moisture distribution with depth at the point r.

Most difficult in the solution of system (11)-(23) is the assignment of the initial soil moisture distribution with depth and over the area. In two areas in the Medvenka River basin, soil moisture observations were made every 10 days. This number of observations is obviously insufficient for reliable assignment of moisture distribution over the area. For this reason we used data on depths to the water table. At points where the ground water was deep (deeper than 100 cm from the soil surface) we assigned the average moisture distribution with depth of measurements in two areas. Otherwise the soil moisture content was assigned in accordance with the distribution typical of the "capillary fringe", which was computed beforehand from (11) for a maximum soil moisture content at the lower boundary (100 cm).

Computations for several heavy rainfalls showed that all the average characteristics for the basin (runoff and precipitation depths), computed from radar and pluviograph data, differ very insignificantly from each other over individual time intervals and for the entire reference period. These differences remain virtually the same for verious intervals of averaging of the amount of precipitation (10, 30, and 60 min).

The infiltration and yield values at individual instants of time depend significantly on the method by which precipitation data are obtained. Figure 2 shows the pattern of water yield for several instants of time for particular areas. We can see that the effect of the two methods of assigning the initial moisture content on runoff is very weak (runoff from all the particular areas is virtually constant for uniform precipitation over the basin, which corresponds to pluviograph data). In that case, the irregularity of runoff formation is produced by the irregularity of precipitation distribution.

The effect of irregular precipitation distribution on the runoff hydrograph at the outlet gaging station is the smaller the greater the transforming properties of the

242

Table 2

Maximum Discharges (liter/sec) Corresponding to Different
Variants of Assigning Precipitation and Different Averaging
Intervals

Dimensional parameter of travel-time curve, hr	Averaging interval, min	Discharges computed from radar data with interpolation		Discharges computed from pluviograph data
		by second method	by first method	
1,67	10	1860	1870	2040
	30	1850	1850	2000
	60	1800	1800	1990
3,33	10	932	931	1030
	30	926	919	1010
	60	900	896	1010

basin (the greater the dimensional parameter of the travel-time curve τ_1). A comparison of the hydrographs computed from pluviograph and radar data showed that even with similar average amounts of precipitation over the basin, maximum discharges differ considerably. Maximum discharges decrease as the interval of averaging over time increases (Table 2).

For this reason one frequently has to use small reference time intervals for more accurate computation of maximum discharges in small rivers. However, investigations by several researchers [5, 9] showed that the errors in the measurement of the amount of precipitation by radar increase with decreasing averaging time. Grauman and Eagleson [9] represent this relation in the form

$$\sigma(t) = f(S, R) t^{-\alpha}, \qquad (14)$$

where σ is the relative standard error; S is the averaging area; R is the distance to the radar set; t is time in hours; and α is a parameter, which equals 0.24, according to the data of Muchnik, and 0.38, according to the data of Wilson [9]. Taking as the average $\alpha = 0.3$ and using relation (14) one can obtain a correction factor for converting the standard errors determined for some values of S, R, and T to errors corresponding to any other value of Δt and the same S and R

$$K(\Delta t) = \left(\frac{\Delta t}{T}\right)^{-0,3}. \qquad (15)$$

Errors in radar measurement of the amount of precipitation during a rainfall have been investigated most completely [4, 5]. The duration of the rainfalls investigated in [4] was about 4 hours. Then, setting $\sigma(T) = 0.24$, in accordance with [6] for S = 25 km², and substituting T = 4 hours into (15), we obtain a relation for determining the standard errors corresponding to any other averaging interval.

$$\sigma(\Delta t) = 0,36 \, \Delta t^{-0,3}. \qquad (16)$$

To investigate the effect of these errors on the accuracy of computation of runoff at the outlet gaging station, we performed several numerical experiments. Into the precipitation measured by radar and

averaged over 10-, 30-, and 60-min intervals, we introduced random errors with a normal distribution. We found that the errors are not correlated in time and that there are no systematic errors, i.e., the mathematical expectations of errors were taken to equal zero for all the averaging intervals. Then the amount of precipitation with an error at each radar station can be determined at any instant of time from the relation

$$\widetilde{P}_i = P_i + \xi_i \sigma \overline{P} \quad (i = 1, 2, \ldots, 15m), \qquad (17)$$

where ξ_i is a random number, simulated on a computer, with a zero norm and dispersion, equal to unity; σ is the assigned standard error, determined from (16) for each averaging interval; P is the average amount of precipitation measured by radar; and m is the number of instants of time for which measurements are available.

Since precipitation data samples are limited, the mathematical expectation and the dispersion of the simulated random errors can differ significantly from the assigned values. With this in mind, we first normalized the simulated errors to the obtain the assigned dispersion and mathematical expectation. In that case, to compute the amount of precipitation with errors, instead of (17) we used the following relation:

$$\widetilde{P}_i' = P_i + \left(\xi_i - |\xi_i| \frac{\sum\limits_{i=1}^{15m} \xi_i}{\sum\limits_{i=1}^{15m} |\xi_i|} \right) \frac{\sigma^2}{\sigma'} \overline{P}, \qquad (18)$$

where σ' is the standard deviation of the simulated errors for a limited series. Computations showed that when random errors are introduced in accordance with (18), the accuracy of determination of total runoff and precipitation depths, averaged over the basin, decreases insignificantly (Table 3). The errors of maximum discharges are somewhat larger. However, in this case also they are much smaller than the simulated errors. The standard errors of precipitation, runoff, and maximum discharges characteristically increase somewhat with the averaging interval. This is apparently associated with the fact that with limited rainfall duration the probability of compensation of precipitation errors of different signs is greater

Standard errors of the relative average precipitation (σ_P), runoff (σ_y), and maximum discharges (σ_Q) over a basin for various averaging intervals

Averaging interval, mm	$\sigma(\Delta t)$	σ_P	σ_y	σ_Q	$\frac{\sigma_y}{\sigma_P}$	$\frac{\sigma_Q}{\sigma_P}$
10	0,53	0,047	0,096	0,101	2,04	2,15
30	0,45	0,069	0,141	0,167	2,05	2,42
60	0,36	0,077	0,164	0,180	2,13	2,34

for smaller than larger intervals. The longer the rainfall duration, the more similar the standard errors in the total amounts of precipitation for various averaging intervals become.

Table 3 shows that the increase in the standard errors of runoff depths and maximum discharges (as compared to the corresponding values for precipitation) is virtually constant for all the averaging intervals.

It should be noted that these results were obtained for some fixed relation between the dispersions of the errors, in accordance with (16), for various averaging intervals of the amount of precipitation. When more extensive radar precipitation data are accumulated and processed, this relation may prove to be different, which will change the results presented here somewhat. However, it can be assumed that automation of measurements and of data processing will most likely lead to an increase in the relation adopted by us between the dispersions of errors for various averaging intervals.

The fact that we disregarded the correlation of the errors in time may prove to be more significant. This may lead to an accumulation of errors in precipitation measurements over small time intervals and to a reduction in the accuracy of runoff computations. It is impossible to estimate the error quantitatively with allowance for the correlation of errors in time, since no mass radar measurements of the amount of precipitation are made at the present time over small time intervals (of about 10 min), which would make it possible to determine the degree of correlation of errors in time.

Use of Radar Measurements of the Amount of Precipitation in Runoff Models with Concentrated Parameters

One of the output values in runoff computations from models with concentrated parameters is the temporal pattern of precipitation values averaged over a basin. At first glance it seems that radar precipitation data are excessive in this case and have no advantage over pluviograph data.

However, we can cite several factors that increase the accuracy of computation of the runoff hydrograph when detailed maps of precipitation, obtained by radar, are used. They include:

1) an increase in the accuracy of determination of the average precipitation depth over a basin;

2) reduction of the reference time interval because of the greater time resolution of radar data;

3) use of actual precipitation distribution functions over the area at each instant of time.

Comparative investigations of the accuracy of precipitation measurements by radar and rain gages showed [4, 6] that the accuracy of determination of average precipitation over a basin by radar for rivers with basin areas less than 5000 km^2 in 35-45% more accurate than by rain gages with the existing density of the precipitation network (one instrument per 1000-2000 km^2). The measurement system at the Central Forecasting Institute makes it possible to increase this difference to 50-60%, according to preliminary data.

If we express the relation between precipitation and runoff in terms of the runoff coefficient, the increase in the accuracy of runoff computation will be the same as for precipitation. If, however, we consider that the runoff coefficient depends on total precipitation in the following manner, for example,

$$K = cP^n,$$

the increase in the accuracy of runoff computation will be even somewhat greater than for precipitation

$$\frac{Y_{rad}}{Y_{pl}} = \left(1 + \frac{\Delta P}{P_0}\right)^{n+1} \approx 1 + (n+1)\frac{\Delta P}{P_0}, \qquad (19)$$

where Y_{rad} and Y_{pl} are the precipitation depths computed from radar and pluviograph data; P_0 is precipitation according to a rain gage; ΔP is the difference between the radar and rain-gage data; and c and n are parameters. From (19) it follows that in this case the improvement in accuracy is the greater, the larger the parameter n.

The averaging interval of precipitation intensity has a strong effect on the magnitude and dynamics of losses. Infiltration is known to decrease with increasing precipitation intensity under otherwise equal conditions. With a large precipitation averaging interval this factor remains virtually unaccounted for and rainfall intensity is greatly underestimated. As a result, the parameters of models determining infiltration cannot be compared properly with experimental data. For example, the parameter of the model of the hydrometeorological Center of the USSR (physically corresponding to the rate of infiltration into saturated soil) can take a value of 0.0002 mm/min [1], whereas the infiltration coefficient even for clayey soils is no lower than 0.001

mm/min, as a rule. Furthermore, to describe the hydrograph (especially maximum discharges) for small rivers ($F < 5000$ km^2) reliably, one has often to select a reference time interval of several hours. This information can be obtained operationally by means of radar and the accuracy of radar precipitation measurement decreases with the time interval. However, from (16) it follows that the loss in accuracy is not very significant with averaging over one- and two-hour intervals. Experience shows that such an interval is sufficient in runoff computations for rivers with basin areas greater than 200-500 km^2.

The third factor can play an important role for runoff models with concentrated parameters, in which theoretical-probabilistic averaging of input values is used. This includes, for example, the model of the Hydrometeorological Center of the USSR [1], in which the following expression was obtained for computing the average infiltration rate over a basin (disregarding evaporation during precipitation):

$$I = I_n \int_{I_n}^{\infty} f(P)\,dP + \int_{0}^{I_n} P f(P)\,dP, \qquad (20)$$

$$I_n = \frac{M[d]}{K_3} + i_0,$$

where $f(P)$ is the distribution density of precipitation probabilities; $M[d]$ is the mathematical expectation of the soil moisture deficit; and K_3 and i_0 are the parameters of the model. When using the data of a thin precipitation network, it is usually assumed that precipitation is evenly distributed over a basin, and instead of (20) one uses the simplified relation

$$I_1 = \begin{cases} I_n & \text{for} \quad \overline{P} > I_n \\ \overline{P} & \text{for} \quad \overline{P} \leqslant I_n, \end{cases} \qquad (21)$$

where $\overline{P}$ is average precipitation over the basin.

Radar data make it possible to construct an empirical distribution density function of precipitation probabilities $f(P, t)$ at each instant of time. Thus, in that case one can use expression (20) to compute the infiltration rate.

The errors in the computation of infiltration from (21) by comparison with (20) will increase with increasing irregularity of precipitation. For example, for an exponential precipitation distribution

$$f(P, t) = \frac{1}{\overline{P}(t)} e^{-\frac{P(t)}{\overline{P}(t)}}$$

one can obtain an analytic expression for the relative errors of infiltration computation from relation (21)

$$\frac{I_1 - I}{I_1} = \begin{cases} 1 - \dfrac{\overline{P}_n}{I_n}\left(1 - e^{-\frac{I_n}{\overline{P}}}\right) & \text{for} \quad \overline{P} > I_n \\[2mm] e^{-\frac{I_n}{\overline{P}}} & \text{for} \quad \overline{P} \leqslant I_n. \end{cases} \qquad (22)$$

We can see from (22) that with an exponential distribution the errors will be determined by the relation between the average precipitation depth and the potential infiltration rate (I_n). The maximum relative error is 0.37 for $I_n = \overline{P}$.

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